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Hypercubic lattice SAW exponents ν and γ: 3.99 dimensions revisited

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Abstract. The self-avoiding walk (SAW) exponents ν and γ are computed over a range of dimensions ($1 \leq d < \infty$) from *exact* expressions for the mean-square end-to-end distance $\langle R_n^2 \rangle$ and the partition function Q_n of SAWs having a limited number of steps, $n \leq 11$. SAW exponents (ν, γ) for *arbitrary* dimension d are estimated by applying standard extrapolation techniques to our direct enumeration data which has been analytically continued to variable dimension. Exponent estimates obtained from continuum theories of self-avoiding paths are compared with the SAW calculations.

1. Introduction

It is well known that the geometrical properties of self-avoiding and random walks exhibit a strong dependence on spatial dimensionality d and that the 'universal' critical behaviour in many systems undergoing phase transitions is intimately related to the geometrical properties of these walks. Specifically, we mention Symanzik's formulation [1] of ϕ^4 field theory in terms of a 'gas' of interacting Brownian paths, Domb's calculation of $O(m)$ lattice spin model properties in terms of interacting SAWs [2] and De Gennes' discovery of an exact relation between the $m \to 0$ limit of the $O(m)$ model and SAWs [3]. The accurate characterization of the geometrical properties of SAWs is consequently a problem with many practical physical applications, besides the rather obvious applications to the solution properties of polymers [4, 5].

Many analytical and numerical studies of self-avoiding walks with nearest-neighbour interactions on a variety of lattices have appeared since the pioneering studies by Orr [6] and Fisher *et al* [7]. Recent works [8] often emphasize the calculation of the SAW 'critical indices' ν and γ, and the 'connectivity constant' μ. Rigorous results include a proof of the existence [9, 10] of μ and the relation [11], $\gamma = 2\nu = 1$, $d \geq 5$. Conformal invariance calculations in $d = 2$ suggest the exact results [12] $2\nu = \frac{3}{2}$ and $\gamma = \frac{43}{32}$.

The study of lattice SAW models has developed in parallel to analytic theories of self-avoiding paths based on $O(m \to 0)$ field theoretic methods [3] or direct formulations in terms of Wiener path-integration [4, 5, 13]. Application of the perturbative Wilson-Fisher ε-expansion method has provided SAW information that is complementary to lattice model studies [5, 13]. Simple dimensional analysis in the continuum theory of

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interesting. Calculations are in progress for ν and γ for theta-point and neighbour-avoiding (NAWs) polymers ($\omega \to -\infty$) in variable dimensions [50].

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